

Formula Sheet

1 Time Value of Money

1.1 Future Value

The future value of x after n periods of growth at (annual) interest rate a compounded m times per year is

$$x(1 + r)^n$$

where $r = a/m$ is the per-period interest rate.

The effective annual interest rate is

$$i = (1 + a/m)^m - 1.$$

The future value of x after t years of growth at annual growth rate d is

$$x(1 + d)^t.$$

1.2 Present Value

In the following, r is the per-period discount rate, d is the annual discount rate, and there are m periods per year.

The present value of y to be received n periods later is

$$y(1 + r)^{-n} = \frac{y}{(1 + r)^n}.$$

The present value of y to be received t years later is

$$y(1 + d)^{-t} = \frac{y}{(1 + d)^t}.$$

The relationship between r and d is

$$d = (1 + r)^m - 1 \quad \text{and} \quad r = (1 + d)^{1/m} - 1.$$

1.3 Present Value: Perpetuities and Annuities

When the discount rate is r per period, an annuity making n payments of C , each one period apart, starting in one period:

$$\frac{C}{r}(1 - (1 + r)^{-n}).$$

Present value of a perpetuity of C per period, starting in one period:

$$\frac{C}{r}.$$

~~2 Bonds~~

~~A coupon payment of a bond with face value F , coupon rate c and m coupon payments per year is~~

$$~~Fc/m.~~$$

~~If the yield (quoted annually) is y for a bond making m coupon payments per year, the corresponding per period discount rate is (because of the yield quotation convention)~~

$$~~r = y/m.~~$$

~~The price of a bond with face value F , coupon rate c , m coupon payments per year, next coupon payment in 1 period, n coupon payments remaining, and yield y is~~

$$~~F(1 + r)^{-n} + \frac{Fc}{y}(1 - (1 + r)^{-n}).~~$$

3 Inflation

When p is a nominal cost that grows at rate h per year, the nominal cost after t years is

$$p(1 + h)^t.$$

When i is an inflation rate and p is a nominal cost occurring at time u , the real cost as measured in time s dollars is

$$p(1 + i)^{s-u}.$$

The real cost, as measured in base- b dollars, of an actual cost A at time t , is

$$R = A(1 + f)^{b-t},$$

where f is the annual rate of inflation. If the actual cost of something at time t is A_t , and its actual cost changes at an annual rate g , then its actual cost at time u is

$$A_u = A_t(1 + g)^{u-t}.$$

The relationship between the inflation rate f , the actual discount rate d_A , and the real discount rate d_R is

$$(1 + f)(1 + d_R) = 1 + d_A.$$